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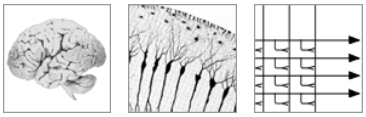

Far-from-equilibrium computation

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Abstract

We study the possibility of developing far-from-equilibrium computational systems, in analogy with corresponding self-organizing physical systems. We review the computational models developed in analogy with physical systems converging towards equilibrium and the research related to the self-organization of far-from-equilibrium physical systems. Based on this review, we define several nonequilibrium computational models. We consider a classification of the state of a nonequilibrium informational system by a linear readout as defining an observer-dependent nonequilibrium steady state. We thus reinterpret the liquid state machine framework as a specific nonequilibrium computational model, which permits its study under new perspectives. We also define two new types of nonequilibrium computational models that may lead to useful applications, similar to the liquid state machine.

1 Introduction

The self-organization of a system is the spontaneous emergence of coherence or of structure, without specific control from outside the system. It is typically the result of numerous interactions between the lower-level components of the system, comprising feedback and non-linearity, and stochasticity (Anderson, 2002; Heylighen, 2003).

Self-organization is a desirable feature of an artificial computational or intelligent system. By self-organizing, a system can learn, perform tasks for which it was not pre-programmed by its designer, and thus display emergent behavior.

Self-organization can be observed in many natural systems, ranging from physical to biological and social systems. Among natural self-organizing systems, physical and chemical ones may be the easiest to analyze, being homogenous: a typical such system is formed by a large number (of the order of 10^{23}) of identical simple interacting elements (molecules, atoms, particles). It is thus interesting to draw inspiration from the study of such systems for the design of analogous artificial self-organizing computational or intelligent systems.

Analogical thinking can often lead to advances in science (Holyoak and Thagard, 1996, Chapter 8). However, one should not forget that partial structural similarities between two different domains do not guarantee the existence of other similarities between the domains (Indurkhya, 1992, Chapter 9). Thus, the transfer of self-organizing principles from physical systems to computational ones may not be straightforward or even possible.

A trivial case of self-organization in physical systems is the evolution of a system towards equilibrium. There exist several computational paradigms that are inspired by this kind of evolution, for example the Hopfield type of neural networks. A more interesting type of self-organization occurs in systems that are far from equilibrium (systems which are subjected to a continuous flux of energy or matter). Computational systems analogous to such systems are relatively less studied.

Our study will investigate the possibility of designing artificial self-organizing computational systems that are inspired by far-from-equilibrium physical systems. First, we will briefly review the self-organization by convergence towards equilibrium and the computational systems it inspired, as an example of transfer of analytical tools and models from physics to computation and artificial intelligence. We will analyze next the phenomenon of self-organization in far-from-equilibrium systems, mentioning examples and reviewing the theoretical analysis of such systems. Several existing nonequilibrium computational systems will be also presented. We will systematically define next possible far-from-equilibrium computational models.

The paper is written in a style that should make it accessible to both physicists and people working in computer science or artificial intelligence.

2 Computational and artificial intelligent systems

2.1 Computational systems

Before embarking in our study, it is useful to define first what we consider to be a computation and a computational system.

In a computation, the computational system is given some informational input, and the system, through its dynamics, generates some informational output. Thus, any dynamical system that depends on an input and has an output that is informationally interesting can be considered a computational system.

The input may be presented to the system at a particular moment of time, followed by the computation, or may be presented continuously during computation. Also, the output can be read at a particular moment of time (at the end of computation), or can be read continuously during computation.

If the input is presented at a particular moment of time, computation follows, and afterwards the output is read, we have a classical computation. If the input is presented once, then the output is read continuously for some time, we have a signal generator, that outputs a signal parameterized by the input. If the input is continuous in time, and the output is discrete in time, we have a system that performs some computation on a signal, or categorizes a signal. If both the input and the output are continuous in time, the system acts as a filter that transforms a signal into another signal.

2.2 Embodied intelligent systems

Genuinely intelligent systems (i.e., adaptive or creative) are necessarily embodied (i.e., they have a body which interacts bidirectionally with an environment, through sensors and effectors) (Florian, 2003). For embodied control systems, the input is the activation of their sensors, and the output is the motor command that will be executed by the effectors. Thus, both the input and the output are continuous in time, and the output should respond in real time to the input.

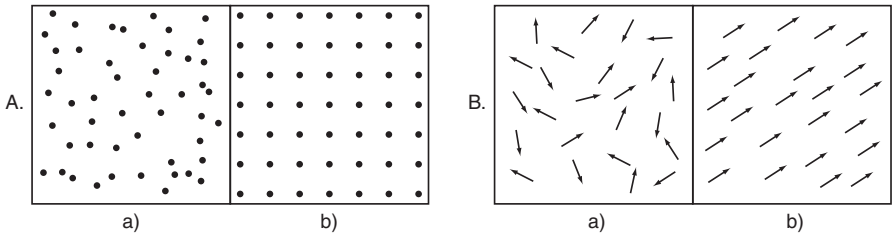

Figure 1: Self-organization by convergence towards equilibrium. A. The regular arrangement of molecules in a crystal: a) disordered state; b) equilibrium state at low temperatures. B. The magnetization of a ferromagnetic substance: a) disordered state; b) equilibrium state at low temperatures.

3 Self-organization by convergence towards equilibrium

The evolution towards equilibrium of some physical systems can be interpreted as a kind of self-organization, as regular structures may result from an initially unstructured state, through interactions between the components of the systems (Heylighen, 2003).

For example, if a particular substance is cooled below a critical temperature, its molecules may arrange in a regular crystalline structure (Fig. 1 A). The structure is the result of the interactions between the molecules. Forces act on the molecules, depending on their relative positions. At close distance, the forces are repulsive, and at long distance, the forces are attractive. Because the molecules stabilize at a distance where the net force acting on them cancels, the forces between identical pairs of molecules are identical, and the system is relatively homogenous, the molecules will be regularly spaced in the crystal. However, the molecules are not always ordered. If the substance is at a high temperature, it may be in a gaseous or liquid state. Such states are characterized by a relatively high disorder, with the molecules constantly moving around and bouncing off one another, randomly.

Another example is the magnetization of a ferromagnetic substance, such as iron (Fig. 1 B). Each iron atom generates a small magnetic field, oriented in a particular direction, proportional to the spin of the atom. At high temperatures, the system is in a disordered configuration, caused by the random movements of the atoms. The spins point in different directions, so that their magnetic fields cancel each other out. When the temperature is lower than a critical value, the spins will spontaneously align themselves, so that they all point in the same direction. The cause of the alignment is the interaction between the spins: two spins tend to rotate one another, such that their directions become parallel. By becoming parallel, they generate a greater magnetic field in that direction, thus orienting more neighboring spins in the same direction.

In both cases, the equilibrium state of the system at low temperature is a highly ordered one, because of the nature of the interactions between components and of their homogeneity.

The systems above are kept, at a particular moment of time, at a given temperature T. At constant temperature, any physical system will evolve towards an equilibrium state that will minimize its free energy $F = E - TS$, where E is the energy and S is the entropy of the system. This macroscopic equilibrium state of minimum free energy corresponds to a particular probability distribution of the microscopic states that are accessible to the system. At zero temperature, the system will evolve to an equilibrium state of minimum energy, which corresponds to a particular microscopic state of the system. The lower the temperature, the closer the states of minimum F will be of the state of minimum E.

Both of the systems described above display phase transitions as a function of temperature, where the macroscopic state of the system changes drastically at certain critical temperatures.

Another example of a physical system converging towards equilibrium is an idealized particle gliding with friction on a landscape of hills and valleys. At finite temperature, the particle is subjected to a random Brownian motion, that allows it to wander across the landscape, preferring however the valleys, where the particle has a lower energy, due to gravitation. At zero temperature, the particle will stabilize at the bottom of the closest valley, where its gravitational energy is the lowest. The self-organization of this system is not so salient as in the previous ones, but, as we will see, its analogous computational systems are more useful.

4 Computation by systems converging towards equilibrium

By looking at the dynamics of systems converging to equilibrium, we may conceive some computational systems with analogous dynamics.

In a first computational model, the input is the initial (nonequilibrium) state of the system, and the output is the final (equilibrium) state of the system, usually at zero temperature (corresponding to the minimum of the energy). We will name this model E_1, for future reference. The computation starts with the presentation of the input, which determines the initial state of the system, and ends when the system reaches equilibrium. The computation results from the natural evolution of the system from an initial nonequilibrium state towards equilibrium. The system might have just one equilibrium state, or multiple equilibrium states, corresponding to local minima of the energy.

We may distinguish two variants of this computational model. In the first one (E_1^a), the equilibrium states, and in particular their structure, are not initially known, and are a relevant outcome of the computation. In the second one (E_1^b), the equilibrium states of the system are known, and the relevant outcome of the computation is an indicator of which of the possible equilibrium states corresponds to the input. The computational system thus establishes a correspondence between classes of inputs and particular outputs.

Many classical algorithms can be seen as analogous to the E_1^a model. For

example, the bubble sort algorithm can be conceived as a process that minimizes the energy in a gravitational field of some particles having a mass proportional to the value to be ordered, constrained to a unidimensional lattice where neighboring particles can exchange places. The equilibrium state of the system is the one with the particles sorted according to their mass, resulting in the corresponding values to be ordered. In general, all optimization algorithms (e.g. genetic algorithms) can be also seen as a convergence to equilibrium, through the minimization of an energy function or the maximization of a fitness function. Any such algorithm specifies dynamical rules that are designed to lead to an optimal state. Given an initial state, the system will lead to a corresponding optimum state, through the intrinsic dynamics of the system, which was designed on purpose. We can thus see that the self-organization of the system is trivial, as its dynamics is predefined by the designer of the system.

The E_1^b model is actually the paradigm of computing with attractors (Amit, 1992; Hertz et al., 1991; Hertz, 2002). An example of such system is the Hopfield type of neural networks, that categorizes the input into predefined categories. These neural networks are fully connected and have symmetric synapses ($w_{ij} = w_{ji}$). The dynamic equations that govern the evolution in time of the neural units, under the condition of synaptic symmetry, permit the existence of a Lyapunov function. The value of this function always decreases during the dynamics of the system, and thus is an equivalent of the energy of a physical system. The states of the system with minimum energy are attractors of the dynamics of the network (neighboring states evolve in time to the attractor) and define basins of attraction in the state space of the network.

The neural network described above can function as an associative memory. The weights of the network can be set such as some of the attractors of the dynamics are the patterns to be stored in memory. Given as input a pattern that is a corrupted version of one of the stored patterns, by setting the network state as the input pattern, the state of the network will evolve to the stored pattern, which is the result of the computation. Because of the analogy with physical systems, concepts and tools from statistical physics were successfully used to draw results such as the storage capacity of such networks, as a function of the number of neural units and the statistical properties of the patterns to be stored.

This kind of networks were studied as a model for understanding the functioning of the brain, but it is currently considered that this model is inappropriate, as there are not sufficient proofs that there are static attractors in the brain. However, these results also had useful applications in artificial intelligence, and thus this computational paradigm can be considered as a successful case of analogical transfer between a physical system and a computational model.

A second computational model (E_2) is the one where the input of the system is the order parameter that controls the phase transitions and the convergence towards minimum energy, analogous to the temperature in the first two physical systems presented in Section 3. The output is a value that indicates the macroscopic state of the system, among predefined possible states. From a computational point of

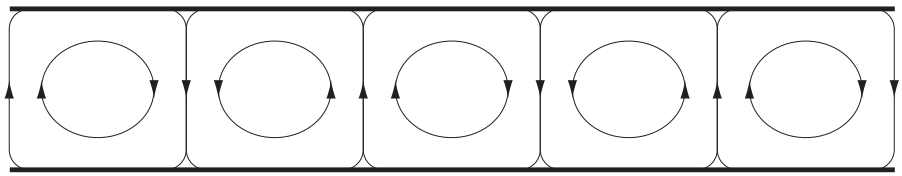

Figure 2: Schematic representation of Rayleigh-Bénard convection showing fluid streamlines in an ideal roll state. The upper surface of the liquid is at temperature T, and the lower one at temperature $T + \Delta T$.

view, the system has a number of metastable states (corresponding to the different phases), and the output will switch between these states, driven by the input.

We are not aware of any practical applications of the E_2 computational model. A reason for the lack of applications might be the fact that the output can have a low, limited number of states, and it depends in a monotonic fashion on the input. Such a simple dependence may be easily coded into a classic algorithm, and an implementation using a system formed from a large number of components, capable of phase transitions, may be superfluous.

In the E_1^b and E_2 computational models, the kind of self-organization that is apparent in the crystallization or the magnetization example, i.e. the apparition of order in a particular state of the system, is not relevant. In these computational models, it is not important that a particular state of equilibrium has a particular structure. The structure might be useful just for recognizing the particular state. However, what matters is that the system has a multitude of equilibrium states, depending on the control parameter or the initial state of the system.

The structure of the state is relevant just for the E_1^a model, but in this paradigm, as we have seen, we can hardly speak about self-organization, as the dynamics of the system is predefined by its designer.

We can thus see that, even for the simple case of self-organization by convergence towards equilibrium, it is not obvious to create useful computational models by analogy with the physical systems. Only the model E_1^b has led to useful developments, and for this model the apparent self-organization of the equilibrium state we have seen in some physical systems is not relevant, but just the fact that the system may choose one of several equilibrium states, depending on the initial state. This conclusion is worth to be taken into account when we will analyze the possibility of creating useful computational systems in analogy with self-organizing far-from-equilibrium systems.

5 Self-organization in nonequilibrium systems

A famous example of self-organization far from equilibrium is the Rayleigh-Bénard rolls (Cross and Hohenberg, 1993). They appear in a thin horizontal layer of viscous liquid heated from below. Thus, there is a positive difference in temperature

ΔT between the lower and the upper surface of the liquid, which generates an energy flux in the system. At low ΔT, the fluid remains at rest, and there is a uniform temperature gradient between the surfaces. The higher temperature of the lower layers dissipate by conduction. At higher ΔT, the warmer, less dense lower layers of fluid tend to rise, a process which is constrained by gravity because the fluid cannot rise as a whole. A convective roll regular pattern appears, where the fluid rises in some regions and falls in others (Fig. 2). This stationary flow is an example of a nonequilibrium steady state (NESS).

This is a non-trivial example of self-organization. The movement of the molecules of liquid is correlated on macroscopic lengths, while their interactions that determine the movement still occur on microscopic scales. The self-organization is determined by the flux of energy, which yields competing interactions in the system, and by the macroscopic properties of the system, like its size, which act as limit conditions.

Other examples of self-organization far from equilibrium are spatio-temporal patterns that appear in reactors during some chemical reactions (such as the Belousov-Zhabotinsky oscillatory reaction or the Brusselator), morphogenesis in living systems, visual hallucination patterns, turbions (Cross and Hohenberg, 1993; Haken, 1989; Prigogine, 1981; Prigogine and Stengers, 1984).

An idealized model that permitted computational and analytical explorations of the properties of nonequilibrium systems is the driven Ising lattice gas (Schmittmann and Zia, 1998; Zia et al., 2000; Kwak et al., 2004; Korniss, 1997).

6 The physics of nonequilibrium systems

Physical nonequilibrium systems are systems which are not isolated, and thus can exchange energy or matter with the environment. They may display a much richer variety of behaviors than equilibrium systems. For example, they may display oscillations, spatial inhomogeneities or patterns, or spatiotemporal chaos. If the driving flux is steady, the systems may converge to a nonequilibrium steady state (NESS), as already exemplified.

Close to equilibrium (in a well-defined sense, i.e., where the forces that drive the system away from equilibrium are small enough that their effects depend linearly of them), the concepts of equilibrium statistical mechanics can still be used for understanding the phenomena involved. Linear irreversible processes like the relaxation processes from nonequilibrium to equilibrium states or the response of a system near equilibrium to weak external disturbances can be studied as an extension of the well-founded statistical mechanics of equilibrium states (Kubo et al., 1985; McLennan, 1989). It is also known that, close to equilibrium, a system will evolve to a steady state characterized by a minimum (but positive, according to the second law of thermodynamics) production of entropy compatible with the constraints imposed to the system. In this framework, the equilibrium appears as a limit case where the production of entropy is zero. As in the steady state the

entropy of the system is constant in time, a positive production of entropy of the system means that the system transfers entropy to the external environment (Prigogine, 1981; Prigogine and Stengers, 1984; Prigogine, 1993).

However, far from equilibrium, quantities like the temperature of the system or other concepts of equilibrium statistical physics like the Gibbs ensemble are defined no more. More importantly, there is nothing analog to the free energy in equilibrium systems, no Lyapunov functional that varies monotonically during the evolution of the system towards the steady states, with a few exceptions. A Lyapunov functional exists just for a few particular far-from-equilibrium systems where it is constructed in a particular fashion (Descalzi et al., 2001; Barré et al., 2002), or for several systems where the steady state corresponds to a maximum production of entropy (Dewar, 2003).

The stability of the nonequilibrium steady state or its independence of fluctuations are no longer guaranteed. In some cases, for certain ranges of order parameters, a steady state may be unstable, and random fluctuations, instead of being averaged out, can be amplified and can invade the entire system, leading it to a qualitatively different macroscopic state. This is a nonequilibrium phase transition, like the one in the Rayleigh-Bénard example above, where, above a critical temperature difference, the conduction steady state changes to the convective one with regular rolls (Prigogine, 1981; Prigogine and Stengers, 1984; Prigogine, 1993; Rácz, 2002). A state of self-organization results, where the system is in a ordered state, that depends on the nonequilibrium macroscopic conditions and the history of fluctuations. A system may have more than one far-from-equilibrium steady states that correspond to a particular set of macroscopic constraints, and thus the actual state of the system depends on its past. The evolution of the system between various steady states, depending on the variation of macroscopic constraints, or its chaotic behavior can be studied in the framework of the bifurcation theory (Prigogine, 1981; Prigogine and Stengers, 1984).

Nobel Prize laureate Ilya Prigogine and physicist Hermann Haken have enthusiastically discussed about the relevance of concepts originated from the study of physical far-from-equilibrium systems for understanding other complex systems, like biological and social ones (Prigogine and Stengers, 1984; Haken, 1989, 1997). Some of Prigogine's writings have also inspired our attempt to apply such concepts to the design of novel computational models. However, at a close look, their claims are exaggerated (Shalizi, 1997, 2002). The theoretical tools currently available, including those developed by Prigogine and Haken, seem to have little applicability outside the particular domains or conditions for which they were developed. Understanding analitically a specific phenomenon, for example nonequilibrium spatiotemporal pattern formation requires specific approximations and perturbative techniques, although these techniques may be useful in the study of more than one phenomenon (Cross and Hohenberg, 1993). In general, it seems that the current theoretical understanding of far-from-equilibrium self-organization is not very advanced. Quoting several specialists: *"In comparison with situations near equilibrium, the behavior of a far-from-equilibrium system becomes as specific as*

possible. There is no universally valid law from which the general behavior of the system to be deducted" (Prigogine and Stengers, 1984)[1]; *"The laws of equilibrium are universal. But far from equilibrium, the behavior can become specific."* (Prigogine, 1981)[2]; *"we do not have a theory even for the NESS"* (Rácz, 2002, p. 4). For example, the properties of the entropy of a far-from-equilibrium system are still not well understood and are the subject of current research (Ruelle, 2003; Goldstein and Lebowitz, 2004; Gallavotti, 2004).

Thus, it is not obvious how a successful transfer of concepts and tools can be made from nonequilibrium physical systems to computational models. We will investigate, however, such possibilities in the following section. The general result that we will retain from physics is that a far-from-equilibrium system is generally not characterized by a Lyapunov function, its behavior can be specific, and thus, among other behaviors, it can self-organize under certain circumstances. More specific results that we identified as potentially useful in the study of nonequilibrium computational systems are some measures of the self-organization of an open system (Klimontovich, 1995, 1997, 1999; Ulgiati and Bianciardi, 1997; Shalizi, 2004), the constraints on transition rates in driven steady states (Evans, 2004), and the slaving principle (Haken, 2004, 1989).

7 Existing nonequilibrium computational models

7.1 The Liquid State Machine

The liquid state machine (LSM) was recently proposed as a new, biologically inspired, nonequilibrium computational paradigm by Maass et al. (2002) and independently by Jaeger (2001) as "echo state" networks. A LSM processes a continuous analog input stream, and its output is also a continuous analog stream. The processing is performed in real time, there is no computation delay other than the delay given by the propagation of the signal through the system. Thus, this kind of system is suitable for the control of embodied agents, or for the analysis of their control system.

The LSM is composed of a nonlinear "liquid" filter L (a function that maps time-varying input streams $u(\cdot)$ to other time-varying streams $x(\cdot)$, also known as operator in mathematics; for example, a random recurrent neural network), and a memoryless readout function f that maps the output $x(t)$ of the filter to a target output $y(t)$ (for example, a simple perceptron). The filter L has a fading memory: $x(t)$ may depend not only on $u(t)$, but also, in a quite arbitrary nonlinear fashion, on previous inputs $u(t-\tau)$. The LSM has universal computational power for time series, if two simple conditions are fulfilled. The first condition is that the filter L separates output-relevant differences in two inputs $u_1(t)$ and $u_2(t)$ to different states

[1] Translated from Romanian edition, I. Prigogine şi I. Stengers (1984), *Noua alianţă: Metamorfoza ştiinţei*, Ed. Politică, Bucureşti, p. 201

[2] Translated from Romanian edition, I. Prigogine (1992), *De la existenţă la devenire: Timp şi complexitate în ştiinţele fizice*, Ed. Ştiinţifică, Bucureşti, p. 93

$x_1(t)$ and $x_2(t)$ (the point-wise separation property). The second condition is that the readout map is able to distinguish and transform different states $x(t)$ into given target outputs $y(t)$ (the approximation property). It was recently shown that, at least for a particular type of neural networks, the computational capabilities of the LSM are largest near the critical boundary where the transition from ordered to chaotic network behavior takes place (Bertschinger and Natschläger, 2004; Natschläger et al., 2004).

The filter L is a dynamical system which is kept out of equilibrium by the input $u(t)$, and thus the LSM is an example of a nonequilibrium computational model. However, the model does not use any theoretical concepts developed in the study of nonequilibrium physical systems.

The initial implementation of the LSM (Maass et al., 2002) used a recurrent spiking neural network as the nonlinear filter L, and a small population of spiking neurons, trained with a perceptron-like local learning rule, as a readout map. However, the nonlinear filter may be implemented also by sigmoid neural networks, either in continuous or discrete time. Many other types of nonlinear filters are possible. Recent implementations included a finite element simulated model of a liquid (Goldenholz, 2003) or even real water in a bucket (Fernando and Sojakka, 2003) for the nonlinear filter, with classic perceptrons for the readout map.

The filter transforms the temporal dynamics of the input stream into a high dimensional spatial pattern. The readout neurons, which may receive inputs from hundreds or thousands neurons from the neural filter, can learn to extract salient information from the high dimensional transient states of the filter, and can transform them to stable readouts. Invariant readout is possible despite the fact that the neural filter may never revisit the same state. The readout map may be linear, and thus extremely easy to train with classic supervised learning methods.

The LSM computational paradigm has certain properties that make it extremely interesting. Multiple readout modules can be trained to perform different tasks on the same neural filter, thus enabling parallel real-time computing. The filter may be a generic circuit, to be used for many tasks, only the readout map needs to be trained for each task. Computational power may be added to the LSM by just adding generic neural circuits to the neural filter, without changing the existing wiring. The system is extremely robust to noise.

7.2 Haken's synergetic computer

Haken (1989, 1997) has developed a model for the so-called "synergetic computer", inspired by some characteristic equations of nonequilibrium pattern formation in physical systems. The synergetic computer performs pattern recognition: the initial state of the system $\mathbf{q}(0)$ is given by the input pattern, and subsequently, through its dynamics, the state $\mathbf{q}(t)$ of the system converges to one of the predefined prototype patterns, $\mathbf{v}^k$, where $\mathbf{q}$ and $\mathbf{v}^k$ are vectors of real numbers. The winning pattern will be the most similar to the initial state $\mathbf{q}(0)$, having the greatest normalized vector product $\mathbf{v}^k \cdot \mathbf{q}(0) \,/\, |\mathbf{v}^k| \, |\mathbf{q}(0)|$.

The dynamics of the system is given by the equation $\dot{\mathbf{q}} = -\nabla_{\mathbf{q}} V(\mathbf{q}, \mathbf{v}^k) + \mathbf{F}(t)$, where $\mathbf{F}(t)$ is a stochastic fluctuating force. Thus, in practice, this system actually evolves to minimize the potential V and thus it is not a far-from-equilibrium system, but a system converging towards equilibrium.

Another approach to nonequilibrium pattern recognition has been developed by Marro et al. (2002).

7.3 Generic neural networks

Any neural network that has a dynamic driven by an input continuous in time is a nonequilibrium system. This kind of networks are often studied in the context of artificial intelligence, but they are seldomly analyzed using theoretical concepts developed in the study of nonequilibrium physical systems.

8 Nonequilibrium systems as computational devices

8.1 General considerations

In the following, we will try to systematically develop nonequilibrium computational models, by analogy with nonequilibrium physical systems. We will first analyze the most important similarities and dissimilarities between a nonequilibrium physical system and an analogous computational one.

Similar to a physical system, a self-organizing computational system should be composed of a large number of relatively simple elements. Their low level interaction, comprising feedback and non-linearity may lead to macroscopic self-organization.

A physical system can be maintained far from equilibrium by fluxes of energy or matter. The corresponding fluxes that keep a computational system in a nonequilibrium state are informational. Whether the input informational fluxes are steady or not may depend on the interpretation of the information they carry.

Physical systems are usually homogenous (e.g., are composed of a large number of identical particles) and are spatially extended. Their self-organization is usually observed as long range spatial correlations (spatial regularities) or temporal oscillations. In contrast, informational systems may need to be highly heterogenous, or to have non-standard topologies, for information processing purposes. For example, computationally interesting neural networks may have a complex structure, midway between full connectivity and nearest-neighbor connections. The complex topology may make difficult the detection of regularities analogous with the spatiotemporal patterns of physical systems, or analytical treatments that need operators analogous to the spatial operators for physical systems. Thus, the lack of intrinsic quantities like space or energy and the complex topology may result in a need for particular measures for assessing the stationarity of the NESS or for their detection. This also suggests that, at least for informational systems, the self-

organization is observer-dependent: regularities in the dynamics of the system may be hidden until the observer discovers them through a particular measure.

A possible solution for these issues is the model of the liquid state machine, which has shown that linear readouts can establish equivalence relationships between dynamic states of high-dimensional nonequilibrium dynamical systems. Thus, nonequilibrium steady (i.e., readout equivalent) states of computational systems can be defined by an observer. Whether the system is or is not in a NESS may depend on the interpretation of the observer, according to her specific purposes.

In contrast to a physical system, a computational system may have, by design, the possibility to adapt to a perturbative input flux by assimilating it. Initially, the flux may drive the system far from equilibrium, possibly up to a NESS. The system may then suffer an internal transformation, adapting to the perturbative flux, resulting a transition from a NESS to a state of equilibrium. Further work is needed to study the properties of such systems.

8.2 Types of nonequilibrium computational models

We may first try to define nonequilibrium computational models analogous to the equilibrium computational ones, where the NESS would be the analogous of the equilibrium state. The similarity between a NESS and a state of equilibrium is that they are both steady, identifiable states. There also is an important difference. For computational systems converging towards equilibrium, we can design the form of the energy landscape in order to have particular states as minima (as in the case of the Hopfield type of neural networks, the E_1^b computational model). For far-from-equilibrium systems, a Lyapunov functional cannot be, in general, defined, and thus we cannot define NESSs by adapting the dynamics of the systems. However, we may define the NESS by adapting the observation of the system's dynamics, as discussed above.

N_1, the analogous of the E_1 computational model, is a system where the input is the initial state, and the output is the final NESS. The flux that keeps the system out of equilibrium does not have here an informational purpose, but is just a parameter of the system. The system should not have a fading memory, like the classic LSM, but should keep the memory of the initial state and reflect it in the resulting steady state. Thus, it should not be neither ordered nor chaotic, but probably at the boundary between chaos and order, like the most efficient LSMs (Bertschinger and Natschläger, 2004; Natschläger et al., 2004).

N_1^a, the analogous of the E_1^a computational model is a system where we define the dynamics according to our purpose, and the relevant output is the structure of the output NESS that corresponds to the input. The dynamics may be defined, for example, by specifying transition rates (Evans, 2004). Further work is needed for analyzing the possibility of implementing this model and its applications.

N_1^b, the analogous of the E_1^b computational model is a system where the possible NESSs of the system are predefined, and the input is categorized by the NESS that results. This may be implemented, for example, by training one or multiple

readouts (as in the LSM model) that would map the NESSs of the system to the desired categories. The multiple NESSs may be studied in the framework of bifurcation theory. Again, further work is needed for analyzing the possibility of implementing this model, its applications and its advantages over the E_1^b model (like, for example, possible faster convergence times).

N_2, the analogous of the E_2 computational model is a system where the input is an order parameter (like the temperature difference for the Bénard rolls), and the output is the resulting NESS. It is not clear whether such a computational system may have any useful applications. Like E_2, the complexity of such a system may be superfluous for such computation that may be achieved more easily within classical computational models.

We are now ready to define two specific nonequilibrium computational models, that have no correspondence within the equilibrium ones. In the N_3 computational model, the input is the informational flux that keeps the system out of equilibrium, and the output is the state of the system (possibly a NESS), or a function of it. This actually corresponds to the liquid state machine computational framework, discussed above (Section 7.1). This computational model is the most interesting compared to the ones already presented, as it allows real-time computation on complex inputs, and is thus appropriate for use as a control system for embodied intelligent agents, or for the analysis of their control systems.

Interpreting the LSM or an intelligent embodied agent as nonequilibrium computational systems allows their study from new perspectives. For example, we may study the states that belong to one category, as categorized by a trained readout, as being a NESS; or we may study them as systems that produce informational entropy.

Our systematic approach, inspired by physical systems, allows us to define a new type of specific nonequilibrium computational model, N_4. The self-organization of far-from-equilibrium physical systems results in long-range spatial correlations, which make the system sensitive to macroscopic spatial constraints. Thus, macroscopic constraints can influence the structure of the NESS. Analogously, we can define a computational model where the input corresponds to macroscopic constraints imposed to the system, while the output is again the state of the system (possibly a NESS), or a function of it.

The N_2, N_3 and N_4 computational models described above can also be mixed, by allowing the input to modify in the same time the order parameters of the system, the informational nonequilibrium flux and the constraints imposed to the system, but probably such a system may be more difficult to analyze and design for useful purposes than the simple models themselves.

The N_3 and N_4 models are specific nonequilibrium models, as they use as computational input the informational flux that keeps the system out of equilibrium. Thus, the informational flux is an intrinsic component of the system, unlike in the other models. For the other models (N_1 and N_2), it may be possible to map the nonequilibrium steady states to equilibrium states, through an informational transformation. Thus, N_3 and N_4 are the most interesting to study.

9 Conclusion

We have reviewed the computational models developed in analogy with physical systems converging towards equilibrium and the research related to the self-organization of far-from-equilibrium physical systems. The equilibrium statistical models have shown that it is not straightforward to design computational systems by analogy with self-organizing physical systems. Out of the four possible models, only one has led to useful developments, and for this model the apparent self-organization of the equilibrium state we have seen in some physical systems is not relevant, but just the fact that the system may choose one of several equilibrium states, depending on the initial state. Also, we have concluded that the current theoretical understanding of far-from-equilibrium self-organization of physical systems is not very advanced, and thus it is not straightforward to design analogous computational systems.

However, this review has led us to define several nonequilibrium computational models. We considered a classification of the state of a nonequilibrium informational system by a linear readout as defining an observer-dependent nonequilibrium steady state. We have thus reinterpreted the liquid state machine framework as a specific nonequilibrium computational model (N_3), which permits its study under new perspectives. We have also defined new types of nonequilibrium computational models (N_1, N_4) that may lead to useful applications, similar to the liquid state machine. Further work will analyze the computational models and their possible implementation.

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